Flexibility of Residential Loads for Demand Response Provisions in Smart Grid

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***Abstract*—One of the main advantages of smart grids is the presence of advanced communication technologies that facilitate grid operators and local distribution companies (LDCs) to communicate directly with customers. While at one end, the end-user customers seek to optimize their energy consumption by deploying home energy management systems (HEMSs), the LDCs on the other hand, seek to improve the grid operational efficiency and reduce losses. In this paper, a two-stage optimization framework is developed wherein multiple HEMS simultaneously optimize their respective energy consumption patterns, and determines their flexibility provision, which is communicated to the LDC. The LDC aggregates the controllable demand profile and the flexibilities of each HEMS to optimize its operational performance and hence the peak reduction signals which are sent to the HEMS. Studies are carried out considering a 33-bus distribution system coordinating with 1295 houses connected at different buses, and with varying customer preferences and objectives to demonstrate the applicability of the proposed scheme.**

***Index Terms*—Demand response, home energy management system, local distribution company, flexibility, smart grid.**

NOMENCLATURE

Indices and Sets

h_j Index for household located at bus j.
i Index for household appliances, $i \in \mathcal{A}$.
j,k Index for bus ($j = s$, substation), $(j,k) \in \mathcal{N}$.
q Index for Energy Storage Device (ESD) and Battery (B).
t Index for time, $t \in \mathcal{T}$.
$\mathcal{H}$ Set of households.

Parameters

C^{FLEX} Objective function weight assigned by LDC on scheduled flexibility.
$E^{min}_{q,h_j,t}$, $E^{max}_{q,h_j,t}$ Minimum and maximum energy level, kWh.
$E^{Initial}_B$, E^{Final}_B Initial and final energy level of battery, kWh.
$E^{AR}_{q,h_j,t}$ Energy level at arrival, kWh.
$G_{j,k}$ Conductance of feeder between buses, $p.u.$
g_{i,h_j} Gas consumption rate, m^3/hr.
$P^{max}_{C_{q,h_j}}$, $P^{max}_{D_{q,h_j}}$ Maximum charging/discharging power, kW.
$P_{D0_{h_j}}$ Connected load in house, kW.
$P^{min}_{g_j}$, $P^{max}_{g_j}$ Minimum and maximum generation level, $p.u.$
$P_{LI_{z,h_j}}$ Rated power of the lighting system, kW.
P_{i,h_j} Rated power of appliance, kW.
$P^d_{j,t}$ Active power drawn by residential loads, $p.u.$
$P^{Imp}_{j,t}$ Total power purchased by LDC from residential loads, $p.u.$
$P^{CAP}_{j,t}$ Peak load of the feeder, $p.u.$
$P^{\overline{FLEX}}_{h_j,t}$ Customer flexibility available, kW.
$P^{\overline{FLEX}}_{j,t}$ Aggregated bus-wise flexibility, kW.
$P_{PV_{h_j,t}}$ PV panel generation in house, kW.
$Q^d_{j,t}$ Reactive power drawn by residential loads, $p.u.$
$t^{AR}_{h_j}$, $t^{DEP}_{h_j}$ Electric vehicle arrival/departure from/to house h_j , min.
V^{min}_j, V^{max}_j Minimum and maximum voltage level, $p.u.$
$Y_{j,k}$ Magnitude of admittance matrix element.
β Peak load factor.
η_1, η_2 Charging/discharging efficiency, %.
ω_{h_j} Charging level factor at departure of household h_j.
ρ^{TOU}_t, ρ^{SEL}, ρ^{GAS} Time of use price $\$/kWh$, selling price to the LDC $\$/kWh$, and Gas price $\$/m^3$.
τ Length of the time interval.
$\theta_{j,k}$ Angle of complex Y-bus matrix element, $p.u.$
Θ^{set}_{i,h_j} Preferred temperature setting of appliance, $^\circ C$.

Variables

$E_{q,h_j,t}$ Energy level, kWh.
$L_{z,h_j,t}$ Illumination level.
$P_{C_{B,h_j,t}}$ Total charging power drawn by Battery, kW.
$P^{PV}_{C_{B,h_j,t}}$ Generated power by PV solar panel used to charge the battery, kW.

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$P^{LDC}_{C_{q,h_j,t}}$	Charging power drawn by device, *kW*.
$P_{D_{B,h_j,t}}$	Total discharged power from Battery, *kW*.
$P^{H}_{D_{q,h_j,t}}$	Discharging power from device to house, *kW*.
$P^{LDC}_{D_{q,h_j,t}}$	Discharging power from device to LDC, *kW*.
$P^{g}_{j,t}$	Active power drawn from substation at bus, *p.u.*
$P^{FLEX}_{j,t}$	Active power flexibility scheduled, *p.u.*
$P^{H}_{LDC_{h_j,t}}$	Delivered power by LDC to meet house demand, *kW*.
$P^{i}_{LDC_{h_j,t}}$	Delivered power by LDC to meet appliance *i* demand, *kW*.
$P^{H}_{PV,h_j,t}$	PV generated power consumed by household appliances, *kW*.
$P^{LDC}_{PV,h_j,t}$	Generated power by PV solar panel sold to LDC in house, *kW*.
$P^{max}_{h_j,t}$	House h_j maximum demand, *kW*.
$\widehat{P}^{max}_{h_j,t}$	Updated house maximum demand, *kW*.
$Q^{g}_{j,t}$	Reactive power drawn from substation at bus, *p.u.*
$Q^{FLEX}_{j,t}$	Reactive power flexibility, *p.u.*
$S_{C_{q,h_j,t}}$	Binary charging status of device, ON/OFF.
$S_{D_{q,h_j,t}}$	Binary discharging status of device, ON/OFF.
$S_{i,h_j,t}$	Binary status of device, ON/OFF.
$V_{j,t}$	Voltage at bus, *p.u.*
$\delta_{j,t}$	Voltage angle at bus, *p.u.*
$\Theta^{in}_{i,h_j,t}$	Temperature level of appliance, °*C*.
$\alpha_{h_j,t}$	flexibility index of customer.

I. Introduction

Effective implementation of Demand Response (DR) programs rely on the availability of smart grid infrastructure, where deployment of two-way communication technologies between the LDC and end-users is very crucial. Smart meters pass information to the LDC on customers' consumption levels and their readiness to participate in DR programs by altering their consumption patterns based on price signals received from the LDC. The LDC can hence achieve a considerable degree of flexible operation by coordinating the controllable loads of retail customers, individually.

The efficient participation of residential customers in DR programs can be achieved using a Home Energy Management System (HEMS) which is a residential controller that seeks to determine the optimal ON/OFF decisions of the household appliances/devices in addition to renewable energy sources (RES) and energy storage devices (ESD) based on the customer's objective function and preferences. This transforms passive residential loads to be active residential energy hubs (REH) [1]. According to a study by Hydro One [2], the display of electricity rates in real time, such as Time of Use (TOU) rates, can increase load shifting on typical days from 3.7% to 5.5%, while on hot days, up to 8.5%, and brings about 13% reduction in energy consumption.

Consequently, several researchers reported the development of HEMS [3]–[5] which optimizes the scheduling of appliance usages to achieve certain objectives of the householder, such as minimization of energy cost. A residential energy management framework was proposed in [3] considering the minimization of both electricity cost and the waiting time of appliances' operation. A mixed integer linear programming (MILP) model was proposed in [4] to optimize the schedule of household appliances, which could be incorporated into the automated residential energy controllers. In [5], a multi-objective mixed integer non-linear programming (MINLP) based HEMS was proposed considering the customer's energy savings and comfort level. However, none of the above works evaluated the customer's flexibility provisions, nor how a system of multiple HEMS could be coordinated such that they could assist the LDC in its operation and provide DR capability.

Some studies have dwelled upon coordinating the residential loads to provide an aggregated DR service to the utility while maximizing the aggregator's benefit.

In [6], an incentive-based energy consumption scheduling problem was formulated, while considering the interaction amongst households and the LDC, to minimize the total energy cost and the peak-to-average ratio of residential loads. Since the work considered only shiftable loads (i.e., dishwasher, washer, and dryer), minimization of energy consumption of the household appliances was not considered. In [7], a decentralized HEMS was formulated as a multi-stage stochastic optimization problem to minimize the aggregator's cost by balancing the residential demand with available generation in real-time; four deferrable loads with fixed load profiles were considered. However, neither [6] or [7] considered the modeling of controllable loads with flexible thermal or electrical capacity such as heating, ventilation and air conditioning (HVAC) and ESD, where energy consumption minimization can be achieved by operating the device within the customer's comfort zone.

In [8], a cooperative energy management system was proposed to coordinate the operation of distributed energy resources (DER) of a neighborhood comprising residential prosumers operating in isolated mode. RES and ESDs were considered, where the energy management system aimed to reduce the mismatch between supply and demand. However, the paper considered the residential loads as fixed loads without taking into account the inherent flexible operation capabilities of the household appliances.

In [9], a centralized coordination approach was proposed to achieve a pre-defined load profile by applying a day-ahead load shifting technique on residential, commercial, and industrial loads. The results indicated that the proposed load shifting approach reduced the grid's peak load and operation cost. Nevertheless, the paper did not investigate the impact of demand management on the grid operations, such as bus voltage levels and system losses.

Distributed heat pumps, which represented a number of households' controllable loads, and battery energy storage system (BESS) were coordinated in [10] to provide power smoothing services to mitigate the impact of power fluctuations of the integrated RES on the connection line between a microgrid and a main grid.

In [11], a decentralized stochastic optimization model was proposed for controlling the energy consumption of several houses including flexible and non-flexible loads, ESD, and RES to minimize the total energy cost of the neighborhood using a real-time scheduling algorithm. The paper considered a case study, where a neighborhood with a few houses was coordinated to minimize the neighborhood cost. The paper assumed that the RES generation surplus would be curtailed without considering a more economical option of selling the generation surplus to the LDC. Besides, the work did not address the impact of the decentralized approach on the distribution grid operation with multiple neighborhoods.

In [12], a decentralized coordination of many prosumers was proposed which aimed to maximize the aggregator's benefit while minimizing the reverse flow of energy at the service point by applying a DR program. Other works in [13] and [14] proposed heuristic approaches to coordinate the DR of residential loads to minimize the cost of each customer, flatten the system load profile, and address the issue of possible peak rebound that might result from uncoordinated DR. The DR of a residential load was achieved by controlling a small set of household appliances- dishwasher, washer, dryer, and plug-in electric vehicle (PEV). Furthermore, the paper investigated the impact of coordinated DR on distribution operation, and an improvement in the voltage profiles and reduction in system losses was noted. However, the work did not consider some of the basic flexible household appliances such as HVAC and stove.

Coordination of a small number of residential HEMS, which comprised controllable appliances in addition to PEV, ESD, and distributed generation to minimize the total neighborhood energy procurement cost while mitigating any unfair use of the distribution transformer, was proposed in [15]. In [16], a two-stage nodal pricing scheme was proposed to motivate the residential customers to participate in DR programs considering distribution system operational aspects; however, power system losses were not considered, and several household appliances were not taken into account in the HEMS as possible flexible loads.

In recent times, flexibility provisions have become a very important topic in power systems as a result of the increasing penetration of RES into the system which calls for an increased number of flexible resources such as controllable loads and ESD. These flexible resources need to be managed wisely to achieve customer-level and grid-level benefits. For example, flexible loads can provide DR services to the LDC. According to [17], uncoordinated DR might affect the distribution grid operation negatively, where proper control of DR services can eliminate the adverse impact of such modifications in the demand-side behavior. Another study stated that the strategic operation of ESD in electricity markets can limit the potential flexibility benefits of such a system [19]. Two-level decomposition approach is proposed in [20] to forecast the available residential load flexibility using smart meters data at the first level and artificial neural network at the second level.

In [18], an electric storage space heater of a residential customer was modeled to optimize its operation considering the flexibility of heating loads while minimizing the total operation cost; on the other hand, the retailer maximized its profit by optimally determining the electricity prices while minimizing the supply-demand imbalance in the system. This was achieved by encouraging the flexible residential customers to alter their consumption behavior in response to the announced prices by the retailer. The study investigated the impact of different pricing schemes on both customers and retailer costs. It was noted that the TOU scheme was the best option for the residential loads. However, it was noted that the retailer model did not consider the distribution grid constraints or the associated power losses, which affects the total cost of the retailer.

Table I presents an overview of the literature on flexibility of residential loads and the ability to provide DR services. It is noted that these works considered different subsets of appliances and home devices, and in many instances they were considered fixed loads. For example, many researchers have merely considered dishwasher, washer, and dryer as flexible loads, while the other household appliances were fixed loads [6], [7], [13]. Other works have mainly investigated the DR provisions from residential thermal loads [16], [18]. Moreover, to the best of the authors' knowledge, very few commercial HEMS solutions exist in the market today that integrate home automation, local generation, and energy storage with the external power system.

The integration of PEV, ESD, and/or RES were considered by a few researchers, which noted the significant changes in the residential load profiles and customers' monthly bills. Very little research has been carried out to explore the impact of residential loads' flexibility and DR provisions on distribution operations [9], [17]. Hence, the inherent benefits of managing flexible resources to facilitate the integration of RES and ESD calls for in-depth research as this subject.

In the present work, a two-stage framework is proposed to aggregate and coordinate the contributions of a large number of REHs to provide flexibility services to the LDC. An REH mathematical model is formulated by significantly improving upon the model in [4], which optimizes the operations of typical home appliances, and ESDs to determine the optimal interactions with the LDC, such as, the power to be sold to or purchased from the LDC, while considering different objectives of the household customer. Thereafter, a residential feeder is considered, where, at each bus, several such REHs, equipped with HEMS are assumed being served by the LDC; each HEMS receives a control signal from the LDC pertaining to a cap on hourly peak load, based on which, each of the HEMSs re-optimizes their residential loads to meet the prescribed hourly load cap. This creates an aggregated DR at each bus, through the load shifting, HEMS optimization, and LDC imposed load cap at the feeder bus. This proposed decentralized approach maintains privacy of individual customers, only requiring them to provide their electricity usage profiles and flexibility to the LDC. Such coordination between the REH and the LDC can improve grid operations by flattening the system load profile, improving bus voltages, reducing losses, and LDC's operating costs. Indeed, the work considers a realistic scenario, where multiple residential customers with

TABLE I
SUMMARY OF LITERATURE REVIEW ON FLEXIBILITY OF RESIDENTIAL LOADS

Paper	REH Model												Grid Operation and Constraints
	Water Heater	Fridge	Stove	HVAC	Lighting	Dishwasher	Washer	Dryer	Pool	ESD	RES	PEV	
[5]	C	F	F	C	NC	C	C	C	NC	C	NC	NC	NC
[6]	F	F	F	F	F	C	C	C	NC	NC	NC	C	Linear feeder model
[7]	NC	NC	NC	NC	NC	C	C	C	NC	NC	NC	C	NC
[8]	F	F	F	F	F	F	F	F	NC	C	C	NC	NC
[9]	NC	NC	C	NC	NC	C	C	C	NC	NC	NC	NC	C
[10]	NC	NC	NC	C	NC	NC	NC	NC	NC	C	C	NC	Tie-line between microgrid and main grid
[11]	Residential loads represented as shiftable and non-shiftable loads									C	C	NC	NC
[12]	Interruptible, uninterruptible, and thremostat-controlled load										C	C	NC
[13], [14]	NC	NC	NC	NC	NC	C	C	C	NC	NC	NC	C	C
[15]	F	F	F	F	F	C	C	C	NC	C	C	C	NC
[16]	C	F	F	C	F	C	C	C	NC	C	NC	C	Linear feeder model
[17]	C	C	C	C	C	C	C	C	NC	NC	NC	NC	C
[18]	C	NC	NC	C	NC	NC	NC	NC	NC	NC	NC	NC	NC
Proposed work	C	C	C	C	C	C	C	C	C	C	C	C	33-bus system

Considered as an optimization variable (C), Fixed Load (F), Not considered (NC)

different behaviors, preferences, and locations, are coordinated to provide a DR capability to the LDC. However, this work does not address the issue of incentives or pricing of flexibility products, and it is assumed that such mechanisms are already in place. Given such arrangements, this paper examines how the residential loads and the LDC interacts and establishes the flexibility provisions to improve distribution system operation.

Based on the above discussion, it may be noted that no works are reported in the literature that evaluate the flexibility from each REH using individually chosen objective criteria, aggregates thousands of such REH flexibilities and integrates them within the LDC's operations model through exchange of inter-linking signals between the REHs and the LDC.

In view of the above discussions, the main objectives of this paper are:

- Extend and augment the mathematical model of the residential customer's HEMS reported in [4] to represent it as an REH considering detailed characteristics of ESDs and photovoltaic (PV) units. The developed model will take into account the detailed interactions, data and information exchanges between different entities within the REH as well as with the LDC.
- Create multiple residential customer profiles, i.e., multiple REHs, each with a unique set of preferences, behavior, and bus locations, connected to a residential distribution feeder. The randomly generated dataset could include individual REH-wise information on customer activity levels, minimum up-time of appliances, preferred usage times, etc. Thereafter, optimize each individual REH to obtain the optimal energy management schedule of the HEMS including dispatch schedules of different devices within the REH.
- Develop a novel concept of REH flexibility index to quantify the flexibility resource available from a household. Thereafter, develop a two-stage coordination scheme that can take into account distinctly individual objectives of each REH as well as that of the LDC, seeking to enhance grid operational efficiency, and hence create an aggregated flexibility service for the system. This will involve a novel LDC operations model which will determine the optimal peak reduction requests from the individual REHs.

Based on the above, the main contributions of the paper are:

- The development of a detailed model of an REH that takes into account a comprehensive set of individual appliance operation models and their optimal operation, considering the customer's preferences, behavior, and objectives. The models of the PV battery and ESD are significantly improved from [4].
- The REH model includes different levels of interactions between several devices internally within the REH and externally with the LDC.
- A novel concept of residential load flexibility is proposed and hence a flexibility index is defined to quantify the available flexibility from a household, aggregated from that provided by various appliances, ESD, and PV panel.
- A novel two-stage coordination scheme and associated novel models of the REH and LDC are proposed that can take into account distinctly individual objectives of each HEMS as well as that of the LDC, seeking to enhance the grid operational efficiency, and create an aggregated flexibility provision for the system.

The rest of the paper is organized as follows: Section II discusses the detailed layout of the residential energy hub coordination scheme between multiple HEMSs and the LDC and

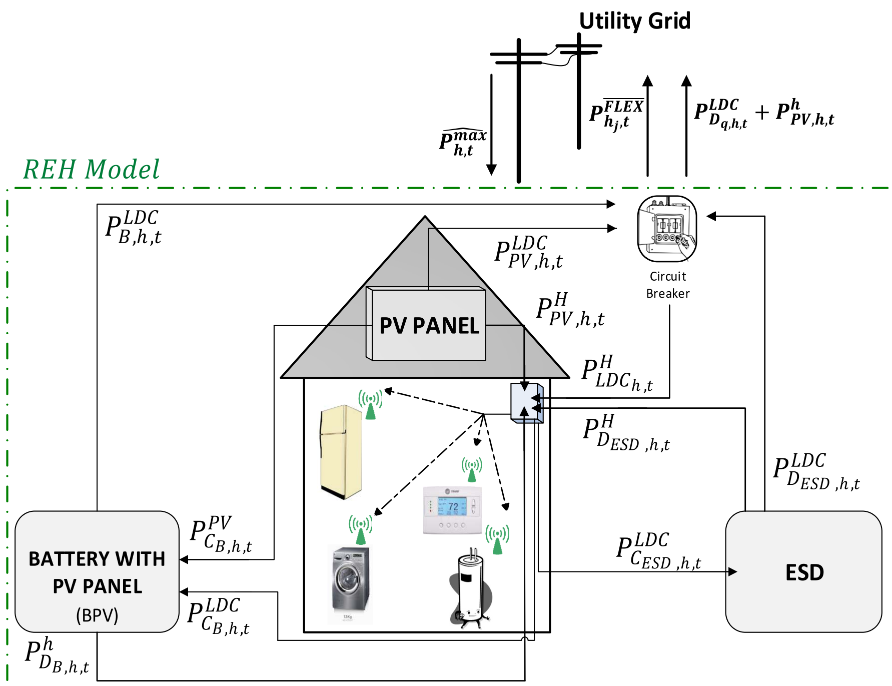

Fig. 1. Schematic of a residential energy hub.

the mathematical models for the HEMS and the LDC operations. The simulation results and the discussion are presented in Section III. The main conclusions of the work are presented in Section IV.

II. Proposed Framework and Mathematical Models

A. Residential Energy Hub and HEMS

The HEMS are residential controllers that carry out scheduling of the REH including all house appliances, the ESD, PV panel battery, and the power interchanges with the external grid, considering the customer's preferences and objectives (Fig. 1). The objective of the HEMS is to minimize energy consumption, energy cost, or maximize comfort level of the customer. A large set of residential customers, all equipped with HEMS is modelled, all households possess the following appliances: fridge, water heater, lighting system, air-conditioning, dishwasher, washer, dryer, stove, and pool. Some households own an ESD and rooftop PV panel (see Fig. 1). The charging and discharging of the ESD are controlled by the HEMS; the ESD is charged by the power drawn from the grid, while at peak demand hours, it can discharge power to the grid and earn revenue. The PV panel battery can be charged either by the power drawn from the grid or from the PV panel directly. The stored energy in the PV panel battery can be used to supply the household or sold to the grid. The HEMS controls the PV panel operation also, i.e., how much is allocated to meet the house load, to charge the PV panel battery, or sell to the grid. Indeed, the ESD and the PV panel battery creates an additional power demand of the household when in charging mode. The PV generation depends the weather conditions and PV panel features. Note that while the PV panel batteries represent stationary storage, the ESDs essentially account for the mobile storage devices present in the house, such as PEV.

B. HEMS Mathematical Model

The HEMS mathematical model is formulated and solved; the model is based on [4], with modifications and several additional features, which are presented below.

1) Objective Functions: Three different objectives are used, for the multiple HEMSs considered in this work, as given below.

Minimize Cost: The objective of REH h_j located at bus j is to minimize its total cost of energy, net of revenue earned.

$$J1_{h_j} = \sum_{t\in\mathcal{T}} \Bigg[\rho_t^{TOU} \Bigg\{ \sum_{\substack{i\in\mathcal{A} \\ i\notin\{LI,ESD,B\}}} P_{i,h_j} S_{i,h_j,t} + \sum_{z\in LI} P_{LI_{z,h_j}} L_{z,h_j,t} + \sum_{i\in\{B,ESD\}} P^{i}_{LDC_{h_j,t}} + P_{D0_{h_j}} \Bigg\} + \sum_{i\in\{HT,WH\}} \rho^{GAS} g_{i,h_j} S_{i,h_j,t} - \rho^{SEL} \Bigg\{ \sum_{i\in\{B,ESD\}} P^{LDC}_{D_{i,h_j,t}} + P^{LDC}_{PV,h_j,t} \Bigg\} \Bigg], \ \forall\, h_j \in \mathcal{H} \quad (1)$$

In (1), the terms within the first curly brackets represents the power drawn by household appliances, the lighting system, the charging power drawn by the ESD and PV panel battery, and the uncontrolled load; which all together, denote the total power drawn from the grid. The term associated with ρ^{GAS}

presents the total cost of gas consumption. The discharged power from ESD and PV panel battery accounts for the power sold to the grid.

Minimize Energy Consumption: The customer minimizes its total energy consumption over the day.

$$J2_{h_j} = \sum_{t\in\mathcal{T}} \Bigg[\sum_{\substack{i\in\mathcal{A} \\ i\notin\{LI,ESD,B\}}} P_{i,h_j} S_{i,h_j,t} + \sum_{z\in LI} P_{LI_{z,h_j}} L_{z,h_j,t} + \sum_{i\in\{B,ESD\}} P^{i}_{LDC_{h_j,t}} + P_{D0_{h_j}} + \sum_{i\in\{HT,WH\}} g_{i,h_j} S_{i,h_j,t} \Bigg], \quad \forall h_j \in \mathcal{H} \quad (2)$$

Maximize Comfort Level: The customer maximizes its comfort level by minimizing the temperature deviations of certain appliances from their preferred set points.

$$J3_{h_j} = \sum_{t\in\mathcal{T}} \Bigg[\sum_{i\in\{AC,HT,WH,FR\}} \left(\Theta^{in}_{i,h_j,t} - \Theta^{set}_{i,h_j}\right)^2 \Bigg], \ \forall h_j \in \mathcal{H} \quad (3)$$

2) Appliance Constraints: The customer h_j appliances operational model was discussed in detail in [4].

Several constraints are modified from [4] to model the inter-relationships between the household, the ESD, and the PV panel (including battery), as presented below.

3) Peak Load Constraint: Ensures that the total load of house h_j is within a limit specified by LDC, $\widehat{P}^{max}_{h_j,t}$.

$$\sum_{i\in\mathcal{A}} P_{i,h_j} S_{i,h_j,t} + \sum_{z\in LI} P_{LI_{z,h_j}} L_{z,h_j,t} + \sum_{i\in\{B,ESD\}} P^{LDC}_{i,h_j,t} - \sum_{i\in\{B,ESD\}} P^{H}_{D_{i,h_j,t}} - P^{H}_{PV,h_j,t} \le \widehat{P}^{max}_{h_j,t}, \ \forall h_j \in \mathcal{H} \quad (4)$$

In (4), the household load includes the appliances' consumption, charging power of ESD and PV panel battery; net of the power discharged from the ESD, the PV panel battery, and that generated by the PV panel to supply some of the household loads.

$$\widehat{P}^{max}_{h_j,t} = P^{max}_{h_j,t} - \alpha_{h_j,t} P^{FLEX}_{j,t}, \quad \forall t \in \mathcal{T}; \forall h_j \in \mathcal{H}; \forall j \in \mathcal{N} \quad (5)$$

Constraint (5) presents the two-way communication between the LDC and the HEMS- the updated maximum demand of house h, $\widehat{P}^{max}_{h_j,t}$, is based on the previous maximum allowable demand, $P^{max}_{h_j,t}$, and a portion of the required flexibility by the LDC at bus j; $\alpha_{h_j,t}$ represents the customer's flexibility index, which is explained in Section II-C.

4) Home Power Balance: Ensures that the total power demand of the household appliances is met by the power drawn from the grid and power discharged to the house from the ESD and PV panel battery, and that generated by the PV system, as given below:

$$\sum_{i\in\mathcal{A}} P_{i,h_j} S_{i,h_j,t} = P^{H}_{LDC,h_j,t} + \sum_{q} P^{H}_{D_{q,h_j,t}} + P^{H}_{PV,h_j,t}, \ \forall t \in \mathcal{T}; \ \forall h_j \in \mathcal{H} \quad (6)$$

5) Energy Storage Device: In [4], the charging level of the ESD was assumed known at each time interval, which is not realistic. The ESD, which are assumed in this work to represent PEV in the household, would be operating within a given charging and discharging time window, as presented below:

$$E_{ESD,h_j,t} = E_{ESD,h_j,t-1} + \tau\Big[P^{LDC}_{C_{ESD,h_j,t}}\eta_1 - \Big(P^{LDC}_{D_{ESD,h_j,t}} + P^{H}_{D_{ESD,h_j,t}}\Big)/\eta_2\Big], \quad \forall t \in \Big\{t^{AR}_{h_j}, t^{DEP}_{h_j}\Big\}; \ \forall h_j \in \mathcal{H} \quad (7)$$

$$E^{min}_{ESD,h_j} \le E_{ESD,h_j,t} \le E^{max}_{ESD,h_j}, \ \forall t \in \Big\{t^{AR}_{h_j}, t^{DEP}_{h_j}\Big\}; \ \forall h_j \in \mathcal{H} \quad (8)$$

$$P^{LDC}_{C_{ESD,h_j,t}} \le S_{C_{ESD,h_j,t}} P^{max}_{C_{ESD,h_j}}, \ \forall t \in \Big\{t^{AR}_{h_j}, t^{DEP}_{h_j}\Big\}; \ \forall h_j \in \mathcal{H} \quad (9)$$

$$P^{LDC}_{D_{ESD,h_j,t}} + P^{H}_{D_{ESD,h_j,t}} \le S_{D_{ESD,h_j,t}} P^{max}_{D_{ESD,h_j}}, \quad \forall t \in \Big\{t^{AR}_{h_j}, t^{DEP}_{h_j}\Big\}; \ \forall h_j \in \mathcal{H} \quad (10)$$

$$S_{C_{ESD,h_j,t}} + S_{D_{ESD,h_j,t}} \le 1, \ \forall t \in \Big\{t^{AR}_{h_j}, t^{DEP}_{h_j}\Big\}; \ \forall h_j \in \mathcal{H} \quad (11)$$

$$E_{ESD,h_j,t} \ge \omega_{h_j} E^{max}_{ESD}, \quad \forall t = t^{DEP}_{h_j}; \ \forall h_j \in \mathcal{H} \quad (12)$$

$$E_{ESD,h_j,t} = E^{AR}_{ESD}, \quad \forall t = t^{AR}_{h_j}; \ \forall h_j \in \mathcal{H} \quad (13)$$

Equation (7) relates the change in energy level of the ESD as affected by the power drawn, and discharged by the ESD to the grid, and to the house. Constraint (8) ensures that the ESD energy level is within the minimum and maximum limits; and limits on charging and discharging power of the ESD are given by (9) and (10), respectively. Constraint (11) ensures that the charging and discharging process does not occur simultaneously. Constraint (12) ensures that the stored energy in the ESD is greater than or equal to a pre-defined minimum value; in addition, the constraint (13) of the ESD defines the energy level within the device at time of arrival $t^{AR}_{h_j}$.

6) Photovoltaic Panel With a Battery ($i = PV, B$): Houses with PV panels are assumed to be equipped with a 3.6 kW PV panel; the modified model with more detailed and realistic architecture is discussed below (Fig. 1).

Solar Panel:

$$P^{LDC}_{PV,h_j,t} + P^{PV}_{C_{B,h_j,t}} + P^{H}_{PV,h_j,t} = P_{PV,h_j,t}, \quad \forall t \in \mathcal{T}; \forall h_j \in \mathcal{H} \quad (14)$$

As shown in (14), the PV panel generates power, and the HEMS optimizes its usage by distributing it to the battery, household, and grid.

Battery:

$$E_{B,h_j,t} = E_{B,h_j,t-1} + \tau\Big[P_{C_{B,h_j,t}}\eta_1 - P_{D_{B,h_j,t}}/\eta_2\Big], \quad \forall t \in \mathcal{T}; \ \forall h_j \in \mathcal{H} \quad (15)$$

$$E^{min}_{B,h_j} \le E_{B,h_j,t} \le E^{max}_{B,h_j}, \quad \forall t \in \mathcal{T}; \ \forall h_j \in \mathcal{H} \quad (16)$$

$$E_{B,h_j,t} = E^{Initial}_{B}, \quad \forall t = 0; \ \forall h_j \in \mathcal{H} \quad (17)$$

$$E_{B,h_j,t} = E^{Final}_{B}, \quad \forall t = 96; \ \forall h_j \in \mathcal{H} \quad (18)$$

Equation (15) presents the inter-temporal changes of energy level of the battery and (16) defines the limits on its energy level. The initial and final energy level of the battery predefined by the The charging and discharging relationships are formulated as follows:

$$P_{C_{B,h_j,t}} = P^{LDC}_{C_{B,h_j,t}} + P^{PV}_{C_{B,h_j,t}},\ \forall t \in \mathcal{T};\ \forall h_j \in \mathcal{H} \tag{19}$$

$$P_{D_{B,h_j,t}} = P^{LDC}_{D_{B,h_j,t}} + P^{H}_{D_{B,h_j,t}},\ \forall t \in \mathcal{T};\ \forall h_j \in \mathcal{H} \tag{20}$$

In (19), the battery can be charged from the PV panel power, or grid power; (20) denotes that the battery can discharge power to the house, or sell to the grid.

$$P^{PV}_{C_{B,h_j,t}} + P^{LDC}_{C_{B,h_j,t}} \leq S_{C_{B,h_j,t}} P^{max}_{C_{B,h_j}},\ \forall t \in \mathcal{T}; \forall h_j \in \mathcal{H} \tag{21}$$

$$P^{H}_{D_{B,h_j,t}} + P^{LDC}_{D_{B,h_j,t}} \leq S_{D_{B,h_j,t}} P^{max}_{D_{B,h_j}},\ \forall t \in \mathcal{T}; \forall h_j \in \mathcal{H} \tag{22}$$

Constraints (21) and (22) represent the maximum charging and discharging power of the battery; the charging and discharging processes occur at different time intervals, as given below:

$$S_{C_{B,h_j,t}} + S_{D_{B,h_j,t}} \leq 1,\ \forall t \in \mathcal{T};\ \forall h_j \in \mathcal{H} \tag{23}$$

The optimal decisions of the HEMS includes the operating decisions of appliances, the energy sold or purchased to/from the utility grid, the charging/discharging decisions of the ESD and the PV panel battery, and the resulting aggregated load profile of the house.

C. Flexibility Definition and Evaluation

One of the main challenges for large-scale integration of home automation is the high initial cost of these devices. In order to render them financially attractive for both the customer and the LDC, there is a need to make provisions for the HEMS to participate in other services, such as that proposed in this paper. The HEMS integrated within an REH converts the homes as flexibility service providers and thereby increases their financial viability. This REH flexibility also significantly improves the distribution system operation.

In the context of the present work, flexibility of REHs is defined as the amount of load variation that is possible by a household at a given time interval, and which can be made available to the LDC for system balancing. Flexibility of REHs can either be positive, indicating a possibility for load reduction by the REH, or negative, indicating possible load increase.

Before the emergence of smart meters and dynamic pricing schemes, such as TOU pricing, customers were charged a flat rate tariff for their electricity consumption; consequently, customers would seek to minimize the total daily energy consumption. Such a load profile of the customer will be referred to as the "base line" demand. With emergence of dynamic pricing and smart meters, customers can now seek to minimize their cost, or maximize their comfort, or choose any other objective criterion. The load profile so obtained, will be referred to, as "desired load profile".

Based on the above definitions, the flexibility available from an REH, at a given time interval, is the difference between the "desired load profile" and the "base line demand" of the customer, given as follows:

$$P^{\overline{FLEX}}_{h_j,t} = \left[\sum_{i\in\mathcal{A}} P_{i,h_j} S_{i,h_j,t}\right]_{J1_{h_j}\ or\ J3_{h_j}} - \left[\sum_{i\in\mathcal{A}} P_{i,h_j} S_{i,h_j,t}\right]_{J2_{h_j}},\ \forall t \in \mathcal{T};\ \forall h_j \in \mathcal{H} \tag{24}$$

$$P^{\overline{FLEX}}_{j,t} = \sum_{h\in\mathcal{H}} P^{\overline{FLEX}}_{h_j,t},\qquad \forall t \in \mathcal{T};\ \forall h_j \in \mathcal{H} \tag{25}$$

$$\alpha_{h_j,t} = \frac{P^{\overline{FLEX}}_{h_j,t}}{P^{\overline{FLEX}}_{j,t}},\qquad \forall t \in \mathcal{T};\ \forall h_j \in \mathcal{H} \tag{26}$$

As noted from (24), it is essentially assumed that a flexible customer can move its demand from its "desired profile" to the "base line" consumption level in order to provide a flexibility service to the LDC. Notwithstanding, it should also be noted that the variation in customer's consumption patterns can lead to different flexibility levels, which is considered in this work by use of different objective functions for different groups of customers, as discussed latter. The notion of flexibility in the context of this problem is not unique, and there is scope for other definitions. The framework for flexibility provisions proposed in this paper is generic and can accommodate other definitions, without any difficulty.

It should be noted that $P^{\overline{FLEX}}_{h_j,t}$ is considered at the house level, and therefore flexibility of ESDs, batteries, or from PV panels are not examined individually, and do not directly appear in the mathematical model, as shown in Fig. 1, denoting the schematic of the REH, where $P^{\overline{FLEX}}_{h_j,t}$ is shown interacting with the utility grid as an aggregated variable, which in effect comprises the above mentioned individual component-level flexibility.

The total flexibility at a bus is the sum of flexibility from all customers connected at the bus, given by, $P^{\overline{FLEX}}_{j,t}$ in (25). The household flexibility index $\alpha_{h_j,t}$ in (26), is obtained by normalizing a customer's flexibility by the total flexibility at that bus. The optimal flexibility requirement, $P^{FLEX}_{j,t}$, is proportionally allocated to each house based $\alpha_{h_j,t}$, as in (5).

D. LDC Operations Model

The LDC carries out a system level optimization, to determine the required flexibility at each bus, $P^{FLEX}_{j,t}$ which is further scaled down to each house to obtain $\widehat{P}^{max}_{h_j,t}$ from (5), using the flexibility index given in (26). The objective of the LDC is to minimize total system losses and a weighted component of the total flexibility use, as given below:

$$J = \sum_{t\in\mathcal{T}} \left[\frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} G_{j,k}\left(V_{j,t}^2 + V_{k,t}^2 - 2V_{j,t}V_{k,t}\cos\left(\delta_{j,t} - \delta_{k,t}\right)\right) + \sum_{j=1}^{N} C^{FLEX} P^{FLEX}_{j,t}\right] \tag{27}$$

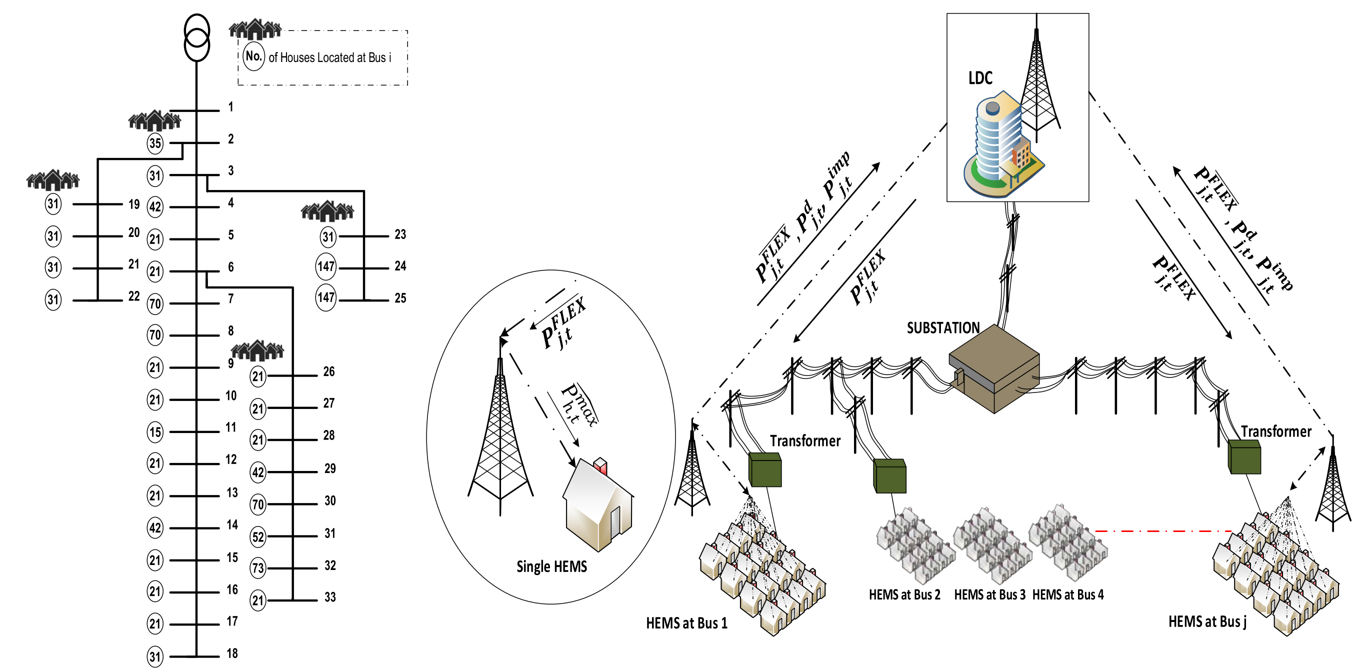

Fig. 2. Schematic of coordination between home energy management systems and local distribution company.

and subjected to the following power flow equations,

$$P^{g}_{j,t} + P^{Imp}_{j,t} + P^{FLEX}_{j,t} - P^{d}_{j,t} = \sum_{k=1}^{N} V_{j,t}V_{k,t}Y_{j,k}\cos\big(\theta_{j,k,t} + \delta_{k,t} - \delta_{j,t}\big), \quad \forall t \in \mathcal{T};\ \forall (j,k) \in \mathcal{N} \quad (28)$$

$$Q^{g}_{j,t} + Q^{FLEX}_{j,t} - Q^{d}_{j,t} = -\sum_{k=1}^{N} V_{j,t}V_{k,t}Y_{j,k}\sin\big(\theta_{j,k,t} + \delta_{k,t} - \delta_{j,t}\big), \quad \forall t \in \mathcal{T};\ \forall (j,k) \in \mathcal{N} \quad (29)$$

where,

$$P^{Imp}_{j,t} = \sum_{h}^{H}\left(\sum_{i\in q} P^{LDC}_{D_{q,h_j,t}} + P^{LDC}_{PV,h_j,t}\right), \quad \forall t \in \mathcal{T};\ \forall j \in \mathcal{N} \quad (30)$$

In (29), $Q^{d}_{j,t}$ and $Q^{FLEX}_{j,t}$ are obtained from $P^{d}_{j,t}$ and $P^{FLEX}_{j,t}$ using constant load power factor of 0.9. $P^{Imp}_{j,t}$ is the aggregate imported by the LDC from all houses located at bus j, as given in (30). And, given that,

$$P^{d}_{j,t} = \sum_{h}^{\mathcal{H}}\left(\sum_{\substack{i\in\mathcal{A} \\ i\notin\{LI,ESD,B\}}} P_{i,h_j}S_{i,h_j,t} + \sum_{z\in LI} P_{LI_{z,h_j}}L_{z,h_j,t} + \sum_{i\in\{B,ESD\}} P^{i}_{LDC_{h_j,t}} + P_{D0_{h_j}}\right), \quad \forall t \in \mathcal{T};\ \forall j \in \mathcal{N} \quad (31)$$

In (31), $P^{d}_{j,t}$ represents the aggregated load at bus j and time t from multiple residential customers. The LDC operations model also includes constraints on bus voltages and power drawn from the substation, as given below:

$$V^{min}_{j} \le V_{j,t} \le V^{max}_{j}, \quad \forall t \in \mathcal{T};\ \forall j \in \mathcal{N} \quad (32)$$

$$P^{min}_{g_j} \le P^{g}_{j,t} \le P^{max}_{g_j}, \quad \forall t \in \mathcal{T};\ \forall j = s \quad (33)$$

E. Coordination of Multiple HEMS and LDC

A novel coordination scheme between the multiple residential customers and the LDC is presented herein.

- It is assumed that all REH are equipped with HEMS which receive input updates such as customer preferences and weather forecast, as shown in Fig. 3 under "Data collection and analysis".
- The REHs are clustered in three groups based on their objective function and their possession of ESD and PV panel. The HEMSs then optimizes the operations, using the optimization models OPT.1, OPT.3, and OPT.5, of their respective household appliances considering one of: cost minimization or comfort maximization objective, and appliance operational constraints, see Fig. 3. Furthermore, the last process at the house level is to evaluate the available household flexibility.
- The resulting optimal load profile and other optimal decisions, such as how much power the house can sell to the distribution grid, and the available flexibility are communicated to the LDC, one day ahead, $P^{Imp}_{j,t}$, $P^{d}_{j,t}$, and $P^{\overline{FLEX}}_{j,t}$ (see Figs. 2 and 3).
- The LDC develops a bus-wise load profile by aggregating the customers' optimized load profiles, executes the LDC operations model, and hence evaluates its system operating conditions considering its objective and constraints (27)-(33). In addition to the mentioned constraints, the LDC can take advantage of the inherent

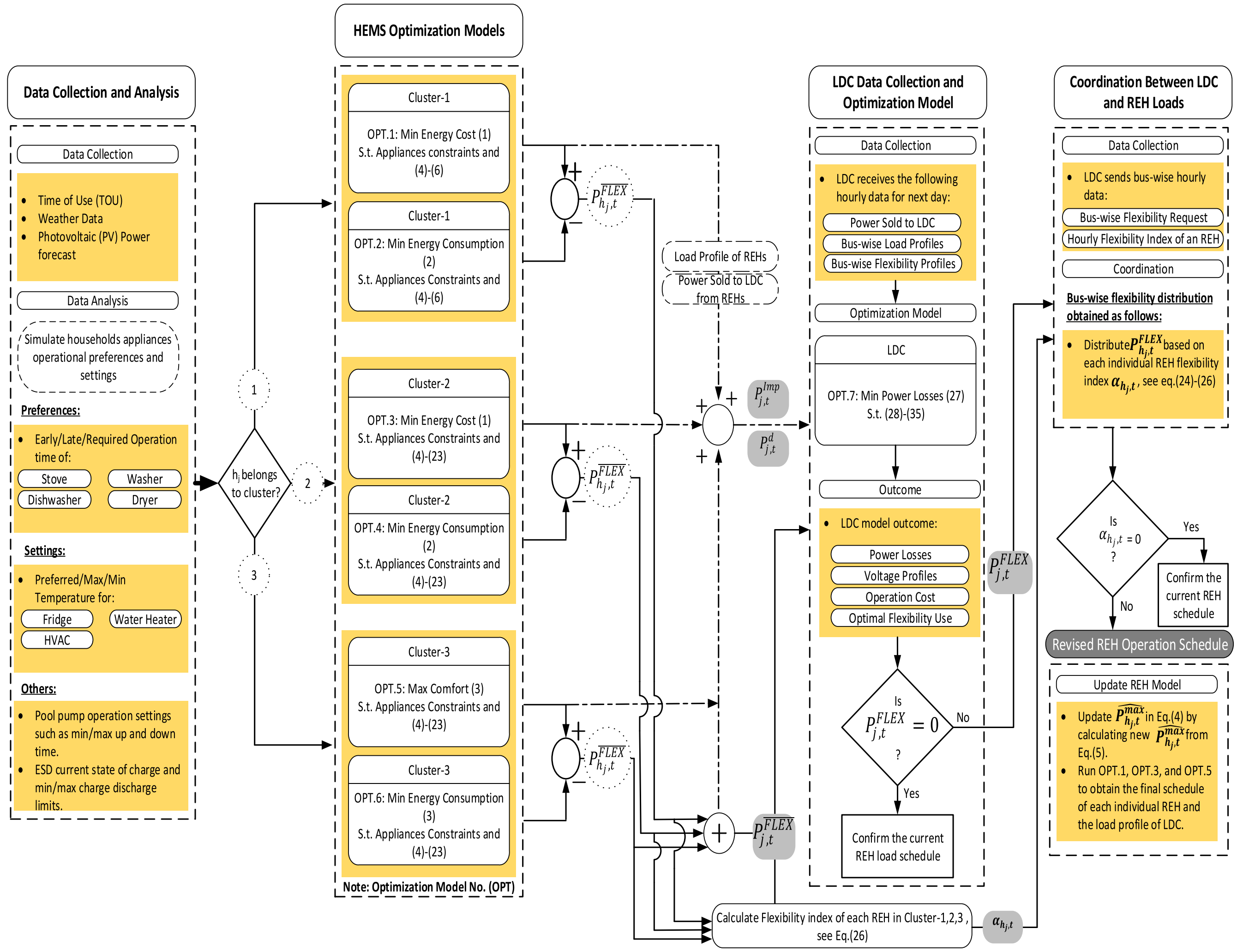

Fig. 3. Coordination of multiple HEMS and LDC.

flexibility of the customers by imposing a peak demand cap. Such decision is limited by the customers' evaluated flexibility, see constraint (34). This cap changes the operation schedules of the participating customers and benefits the LDC. The peak load of the feeder at bus j, hour t, is $P^{cap}_{j,t}$; when the LDC has to request for flexibility from customers, it sends a request signal $P^{FLEX}_{j,t}$, as shown in Fig. 2; with an appropriate choice of β, where $P^{FLEX}_{j,t}$ is limited by the aggregated customers' flexibility, at bus j, as given below:

$$\beta P^{cap}_{j,t} \leq P^{FLEX}_{j,t} \leq P^{\overline{FLEX}}_{j,t} \quad if \quad P^{\overline{FLEX}}_{j,t} \geq 0, \ \forall j \in \mathcal{N}$$
$$-\beta P^{cap}_{j,t} \geq P^{FLEX}_{j,t} \geq P^{\overline{FLEX}}_{j,t} \quad if \quad P^{\overline{FLEX}}_{j,t} \leq 0, \ \forall j \in \mathcal{N} \quad (34)$$

In (34), $P^{\overline{FLEX}}_{j,t}$ positive denotes that the customers are willing to reduce their consumption at bus j and time t, from their optimal schedule to the minimum energy consumption operation. While, $P^{\overline{FLEX}}_{j,t}$ negative indicates the customer is willing to increase its consumption.

- From this analysis, the hour-wise optimal flexibility use required by the LDC is determined considering the bus-wise flexibility obtained from (24)-(26).
- This bus-wise, hourly, optimal flexibility is then proportionally allocated to each house based on the flexibility index $\alpha_{h_j,t}$ of the customer, and communicated to individual HEMS. The respective HEMS in turn incorporates these signals as a 'peak demand cap' (5), and re-optimizes their respective appliance schedules.
- Note that, a household will only submit to the LDC its aggregate load profile, how much it would sell/buy to/from the grid, and how much flexibility provision is available next day. There will be no exchange of private information of the customer or its activity schedules.
- The HEMS and the LDC optimization models are interlinked by external signals, as shown in Fig. 2, wherein the LDC receives the $P^{Imp}_{j,t}$, $P^{d}_{j,t}$, and $P^{\overline{FLEX}}_{j,t}$ signals from HEMS while the HEMS receives $P^{FLEX}_{j,t}$ signal from the LDC, based on which the optimal decisions of each, are determined. This is a two-stage sequential process involving information exchange

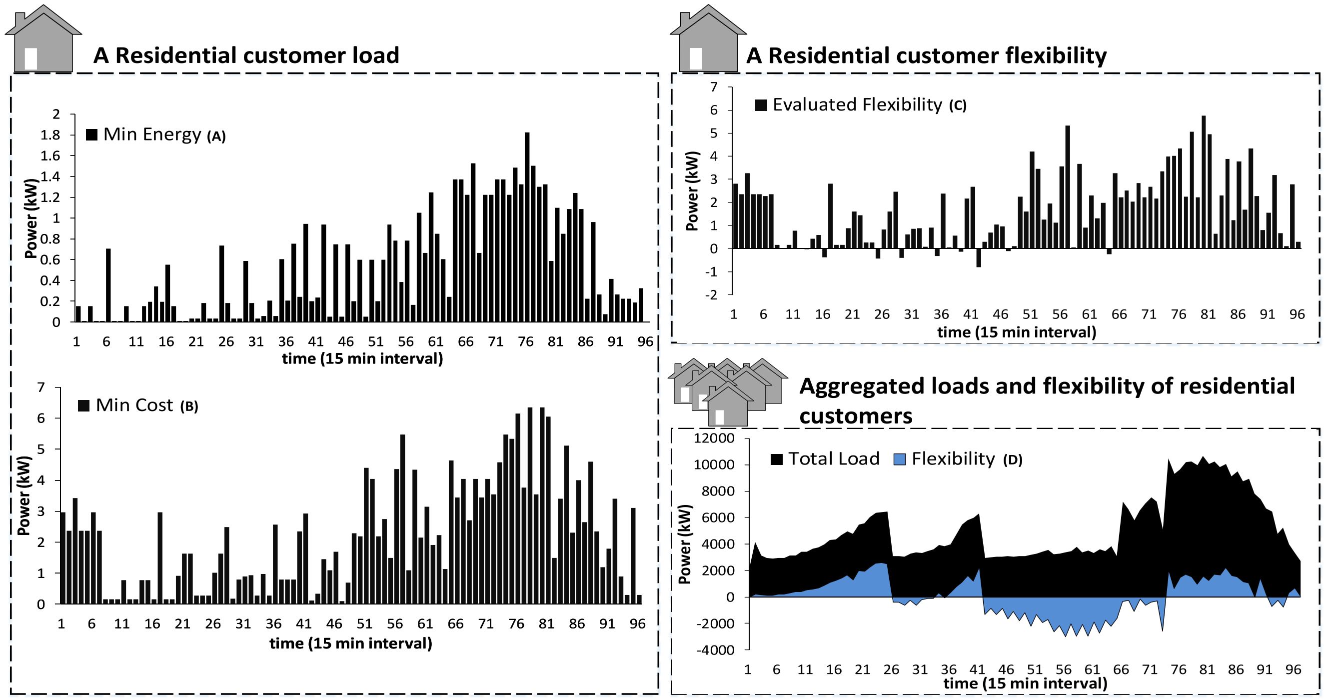

Fig. 4. Load and flexibility profiles at residential and grid levels.

and effective communication between the houses and the LDC.

The proposed coordination scheme enables each REH to carry out its desired operation based on its own objective while preserving information privacy and security while at the same time providing a flexibility service to the LDC to improve grid operations and reliability.

III. Results and Discussions

A. Case Study

In this work, the LDC is responsible for managing a 33-bus distribution system and meeting the demand of 1,295 houses connected at its various buses. The number of houses at each bus is shown in Fig. 2 (within the circles), which is obtained assuming that the connected load at a house is 7 kW. In order to represent each HEMS individually, unique objective functions, activity levels, and customer preferences (i.e., the required operation time and minimum up/down time of washer, dryer, stove, etc.) are randomly generated. Furthermore, the TOU rates applicable in Ontario, Canada, are used in this study [21]. The list of parameters and their values, associated with a single typical HEMS model is provided in the Appendix.

Three customer clusters are considered to evaluate the flexibility of residential customers in providing DR services to the LDC: Cluster-1: 80% of the customers own typical household appliances and seek to minimize their energy cost; Cluster-2: 10% customers own typical household appliances, ESD, and PV panel, and seek to minimize their energy cost; and Cluster-3: 10% customers own typical household appliances, ESD, and PV panel, and seek to maximize their comfort level. Each cluster of customers are distributed at all buses uniformly. It is important to mention that many households do not change their initial consumption level either because of their limited flexibility in consumption patterns or their limited affect on the LDC system operation. The customers from Cluster-2 and -3 do not impact the system peak load since they own RESs which reduce their energy consumption during peak hours. Consequently, the results of Cluster-1 customers are only discussed in this paper. Different peak reduction required (PRR) values have been considered to evaluate the customers flexibility, as follows:

- Case-1: Base case, no PRR requested by LDC, $\beta = 1$;
- Case-2: PRR request by LDC is 5%, *i.e.*, $\beta = 0.95$;
- Case-3: PRR request by LDC is 10%, *i.e.*, $\beta = 0.9$;
- Case-4: PRR request by LDC is 20%, *i.e.*, $\beta = 0.8$.

B. HEMS Level Outcomes

At the house level, Table-II presents one typical household's appliance-wise consumption on a summer day (house #400 located at bus-14). Figure 4 (A) and (B) shows a residential customer's load profile while minimizing energy and energy cost, respectively. In addition, it presents the customer flexibility in Fig. 4 (C), and this information is communicated to the LDC.

It can be noted that the energy cost rate of the household increases from Case-I (Base Case) to the cases with flexibility provisions, as a result of the PRR imposed by the LDC, which limits the power that can be drawn from the grid at low price hours. Fig. 5 shows the temperature deviation inside house #400, refrigerator and the operation of the water heater for different values of β (different peak caps) imposed by the LDC. Note that the range specified by the customer for the

TABLE II
OPERATIONAL SUMMARY OF HOUSE #400 PER DAY, FOR DIFFERENT $P_{j,t}^{PRR}$

Device		CASE 1 (Base)		CASE 2 (5% PRR)		CASE 3 (10% PRR)		CASE 4 (20% PRR)	
		(kWh)	(\$)	(kWh)	(\$)	(kWh)	(\$)	(kWh)	(\$)
Fridge		1.425	0.501	1.463	0.520	1.425	0.522	1.425	0.517
Water Heater	Elec.	0.225	0.078	0.225	0.078	0.225	0.078	0.225	0.078
	Gas	0.111	0.013	0.111	0.013	0.111	0.013	0.111	0.013
Lighting		1.245	0.453	1.245	0.453	1.245	0.453	1.245	0.453
AC		5.225	1.855	5.363	2.038	5.363	2.001	5.225	1.982
Dishwasher		0.350	0.091	0.350	0.131	0.350	0.161	0.350	0.142
Washer		0.225	0.059	0.225	0.074	0.225	0.100	0.225	0.089
Dryer		0.555	0.144	0.555	0.181	0.555	0.262	0.555	0.225
Stove		1.125	0.293	1.125	0.393	1.125	0.483	1.125	0.497
Pool		1.875	0.538	1.875	0.641	1.875	0.627	1.875	0.614
Total		12.361	4.024	12.536	4.522	12.499	4.700	12.361	4.608
Energy Cost Rate (\$/kWh)		0.325		0.361		0.376		0.372	

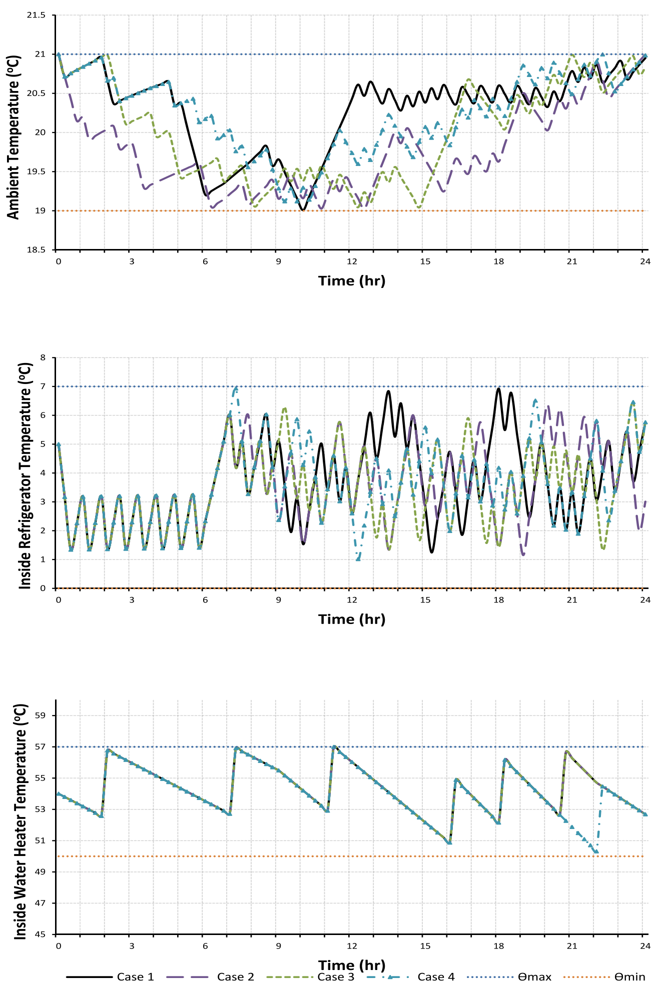

Fig. 5. Temperature profile inside house #400, refrigerator, and water heater.

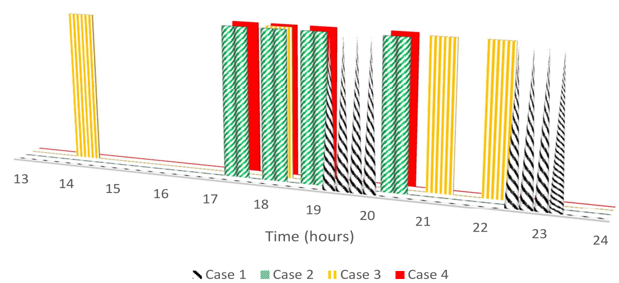

Fig. 6. Optimal operation of selected appliance (dishwasher) at house #400.

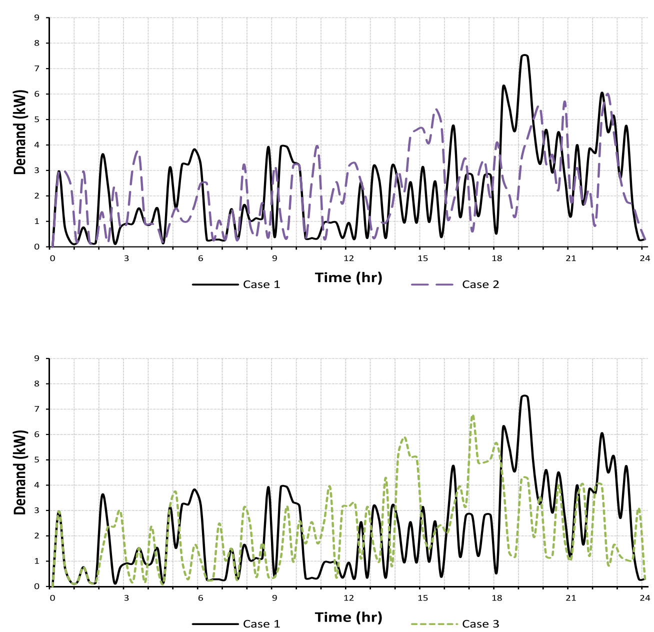

Fig. 7. Optimal load profile for house h_j at different PRR requests.

minimum and maximum temperature, governs the degree of customer flexibility that would be available.

Fig. 6 shows the ON/OFF operation of the dishwasher in house #400, which depends on the required operation time (i.e., eight time intervals), operation interval (i.e., from 2 PM to midnight), and the minimum time gap between two consecutive operations (minimum of two time intervals). In Case-1 (Base Case), the operation of the dishwasher is scheduled at two periods, each of four time intervals. It can be noted that

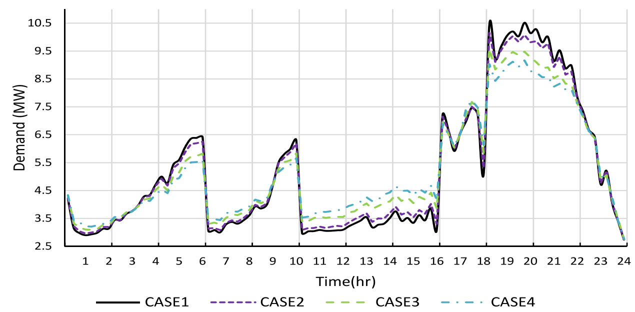

Fig. 8. Total load profile of LDC.

TABLE III
LDC Load Profile Characteristics and Power Losses

Case	Minimum Load (kW)	Peak Load (kW)	Average Load (kW)	Power Losses (Rate of Change%)
Case 1 (0% PRR)	2722.1	10515.78	5175.45	—
Case 2 (5% PRR)	2735.5 (0.49%)	10150.97 (-3.47%)	5178.01 (0.05%)	-1.341%
Case 3 (10% PRR)	2754.7 (1.2%)	9580.18 (-8.9%)	5187.2 (0.23%)	-6.124%
Case 4 (20% PRR)	2765.7 (1.6%)	9164.81 (-12.85%)	5198.2 (0.44%)	-17.830%

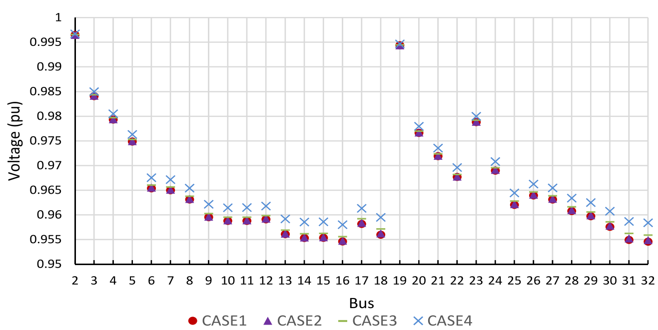

Fig. 9. Average voltage at each bus at different PRR requests.

the optimal schedule changes with different peak caps signals (β values) imposed by the LDC.

Fig. 7 illustrates the variation in the total household consumption between Case-1 and Case-2; and Case-1 and Case-3, respectively. It is noted that the peak hours of power usage of the house is shifted as a result of peak signals sent by the LDC.

C. LDC Level Outcomes

At the LDC side, Fig. 8 presents the total load profile of the distribution system. It can be noted from the figure that TOU tariffs, which are applicable in the present work, causes peak rebound effects at hours 19-21, which means that a peak demand is created when the TOU price reduces to the off-peak price levels at hour 19. In order to circumvent this situation, the LDC utilizes the available flexibility, inherent in the customers' consumption patterns (i.e., as shown in Fig. 4), to improve the total load shape and reduce the peak demand at those hours.

Table III illustrates the improvements in the total load profile of the LDC and system losses, for the different cases considered. It is important to note that the proposed coordination scheme not only shifts the peak load, but it also reduces it, and increases both the minimum and the average load of the distribution system, thus flattening the load profile. It is assumed that the LDC requested for a reduction in the peak load between 6 to 10 PM. As noted from Table III, there is a significant reduction in the peak load while the shifted controllable loads increase the total average load of the system. Moreover, the inherent flexibility of the customers reduces the system losses as well. Fig. 9 shows an improvement in the average voltage profile at each bus over the day, when the proposed DR scheme is applied, and justifies the usefulness of the proposed coordination scheme toward efficient grid operations.

TABLE IV
Model Statistics

	House Level					Grid Level
	OPT.1	OPT.3	OPT.5	OPT.2	OPT.4-6	OPT.7
Model Type	MILP					NLP
Solver Type	CPLEX					SNOPT
Single Variables	2,223	3,857	2,223	2,223	3,857	19,203
Single Equations	2,403	3,638	2,400	2,442	3,553	9,527
Discrete Variables	962	1,328	952	992	1,344	
Resource Usage, s	0.2	0.254	0.134	0.188	0.232	8.6
Absolute Gap	7	3	7.5	6	9	
Relative Gap	0.09	0.05	0.09	0.07	0.09	

*Optimization Model (OPT)

D. Computational Aspects

The proposed REH model is an MILP problem which is solved using CPLEX solver; the LDC operations model is a nonlinear programming (NLP) problem, solved using SNOPT solver, in both cases on the GAMS platform [22]. The optimization models are executed on a Dell PowerEdge R810 server, Windows 10 Education 64-bit operating system, with 4 Intel(R) core(TM) i7 processors and 16 GB of RAM. The individual HEMS optimization programs are executed in parallel by individual customers, and each require less than 30 seconds to solve. Different optimization models are assigned for each individual HEMS, see Fig. 3. Table IV presents an overview of the model statistics of the HEMS and LDC model.

The proposed coordination scheme can be implemented in real-time or day ahead, without any difficulty. It is important to note that the maximum amount of PRR that can be requested by the LDC, can vary based on the customers' willingness to change their consumption patterns. In fact, some customers do not influence the grid operation, so the LDC does not impose such a peak cap to limit the flexibility of their energy consumption.

IV. Conclusion

This paper proposed a novel, two-stage coordination scheme between multiple HEMS and the LDC in order to enhance the grid operational efficiency, reduce peak demand, minimize the power losses; as well as reduce customers' energy cost rate. At the residential customers level, each individual HEMS optimizes the household appliances schedule based on pre-defined requirements and goals by the household.

In addition, each customer's inherent flexibility is evaluated using the proposed approach, which is aggregated with other neighbourhood households to provide flexibility services to the LDC. Bus-wise aggregated loads, power sold to the LDC, and total available flexibility from households, are communicated to the LDC system operator, who evaluates its system operation and determines the bus-wise PRR. Each customer is sent a peak cap signal by the LDC.

The proposed two-stage coordination scheme captures the architecture of real-time implementation of procurement of flexibility provisions by the LDC. Another advantage of the two-stage coordination scheme is the disaggregation and distribution of the computational burden amongst each entity, as compared to that in a centralized optimization model which would need to optimize thousands of REHs as well as the LDC's operations simultaneously.

This work does not consider the incentives that may be provided by the LDC to participants, which remain a subject for future work. The work also does not consider uncertainties of renewables or customer activities, which is left as a topic for future studies. The results from the proposed framework demonstrated considerable improvement in the total load profile of the distribution system and voltage level at each bus in the system, while maintaining the desired comfort level of each customer.

Appendix

Table V presents the parameters of appliances with thermal capacity such as fridge, water heater, and HVAC system. Tables VI and VII present the parameters of deferrable appliances, dimmable lighting system, and appliances with energy capacity capability. The weather forecast used for the present studies were taken from [23] for a typical day of Summer 2017 in Ontario. The nomenclature of the listed parameters is presented below:

Θ^0 Initial temperature.
Θ^{PRF} Preferred temperature setting, °C.
Θ^{min} Minimum allowable temperature, °C.
Θ^{max} Maximum allowable temperature, °C.
Ψ Cooling/Warming effect associated with ON state of a device.
Φ Cooling/Warming effect associated with OFF state of a device.
Ω Activity level effect.
P Rated power, *kW*.
t^{ET} Early operation time.
t^{LT} Late operation time.
t^{RT} Required operation time.
t^{MUT} Minimum up-time.
t^{MDT} Minimum down-time.
E^0 Initial energy level, *kWh*.
E^{min} Minimum allowable energy level, *kWh*.
E^{max} Maximum allowable energy level, *kWh*.
O^{GAP} Maximum allowed time gap between operation of washer and dryer.

TABLE V
PARAMETERS OF APPLIANCES WITH THERMAL CAPACITY

Parameter	Fridge	Water Heater	HVAC System
Θ^0	3	50	20
Θ^{PRF}	1	50	21
Θ^{min}	0	48	18
Θ^{max}	5	58	23
Ψ	2.75	4.4	0.67
Φ	0.605	0.068	0.044
Ω	0.14	0.05	0.0075
P	0.6	0.6	2.2

TABLE VI
PARAMETERS OF SELECTED APPLIANCES

Parameter	Stove	Washer	Dryer	Dishwasher	lighting	Pool
t^{ET}	65	64	64	65	1	29
t^{LT}	88	92	92	92	96	96
t^{RT}	12	8	8	8		40
t^{MUT}	4	8	8	8		1
t^{MDT}		3	3	4		1
O^{GAP}			8			
P	1.5	0.45	1.1	0.7	0.15	0.75

TABLE VII
PARAMETERS OF DEVICES WITH ENERGY CAPACITY

Parameter	Energy Storage Device (ESD)	Battery (B)
t^{ET}	65	1
t^{LT}	88	96
E^0	1.5	2.4
E^{min}	1	1
E^{max}	3.6	3.6
P_C^{min}	0.45	0.45
P_C^{max}	0.45	0.45
P	1.5	0.45

References

[1] B. P. Koirala, E. Koliou, J. Friege, R. A. Hakvoort, and P. M. Herder, "Energetic communities for community energy: A review of key issues and trends shaping integrated community energy systems," *Renew. Sustain. Energy Rev.*, vol. 56, pp. 722–744, Apr. 2016.
[2] N. Ruiz, I. Cobelo, and J. Oyarzabal, "A direct load control model for virtual power plant management," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 959–966, May 2009.
[3] A. H. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 120–133, Sep. 2010.
[4] M. C. Bozchalui, S. A. Hashmi, H. Hassen, C. A. Canizares, and K. Bhattacharya, "Optimal operation of residential energy hubs in smart grids," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 1755–1766, Dec. 2012.
[5] A. Anvari-Moghaddam, H. Monsef, and A. Rahimi-Kian, "Optimal smart home energy management considering energy saving and a comfortable lifestyle," *IEEE Trans. Smart Grid*, vol. 6, no. 1, pp. 324–332, Jan. 2015.

[6] A.-H. Mohsenian-Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, “Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid,” *IEEE Trans. Smart Grid*, vol. 1, no. 3, pp. 320–331, Dec. 2010.
[7] T. H. Chang, M. Alizadeh, and A. Scaglione, “Real-time power balancing via decentralized coordinated home energy scheduling,” *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1490–1504, Sep. 2013.
[8] A. C. Luna, N. L. Diaz, M. Graells, J. C. Vasquez, and J. M. Guerrero, “Cooperative energy management for a cluster of households prosumers,” *IEEE Trans. Consum. Electron.*, vol. 62, no. 3, pp. 235–242, Aug. 2016.
[9] T. Logenthiran, D. Srinivasan, and T. Z. Shun, “Demand side management in smart grid using heuristic optimization,” *IEEE Trans. Smart Grid*, vol. 3, no. 3, pp. 1244–1252, Sep. 2012.
[10] D. Wang *et al.*, “A demand response and battery storage coordination algorithm for providing microgrid tie-line smoothing services,” *IEEE Trans. Sustain. Energy*, vol. 5, no. 2, pp. 476–486, Apr. 2014.
[11] Y. Guo, M. Pan, Y. Fang, and P. P. Khargonekar, “Decentralized coordination of energy utilization for residential households in the smart grid,” *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1341–1350, Sep. 2013.
[12] G. Brusco, A. Burgio, D. Menniti, A. Pinnarelli, and N. Sorrentino, “Energy management system for an energy district with demand response availability,” *IEEE Trans. Smart Grid*, vol. 5, no. 5, pp. 2385–2393, Sep. 2014.
[13] A. Safdarian, M. Fotuhi-Firuzabad, and M. Lehtonen, “A distributed algorithm for managing residential demand response in smart grids,” *IEEE Trans. Ind. Informat*, vol. 10, no. 4, pp. 2385–2393, Nov. 2014.
[14] A. Safdarian, M. Fotuhi-Firuzabad, and M. Lehtonen, “Optimal residential load management in smart grids: A decentralized framework,” *IEEE Trans. Smart Grid*, vol. 7, no. 4, pp. 1836–1845, Jul. 2016.
[15] N. G. Paterakis, O. Erdinc, I. N. Pappi, A. G. Bakirtzis, and J. P. S. Catalao, “Coordinated operation of a neighborhood of smart households comprising electric vehicles, energy storage and distributed generation,” *IEEE Trans. Smart Grid*, vol. 7, no. 6, pp. 2736–2747, Nov. 2016.
[16] B. Moradzadeh and K. Tomsovic, “Two-stage residential energy management considering network operational constraints,” *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2339–2346, Dec. 2013.
[17] K. Mckenna and A. Keane, “Residential load modeling of price-based demand response for network impact studies,” *IEEE Trans. Smart Grid*, vol. 7, no. 5, pp. 2285–2294, Sep. 2016.
[18] O. Kilkki, A. Alahaivala, and I. Seilonen, “Optimized control of price-based demand response with electric storage space heating,” *IEEE Trans Ind. Informat*, vol. 11, no. 1, pp. 281–288, Feb. 2015.
[19] K. Hartwig and I. Kockar, “Impact of strategic behavior and ownership of energy storage on provision of flexibility,” *IEEE Trans. Sustain. Energy*, vol. 7, no. 2, pp. 744–754, Apr. 2016.
[20] J. Ponoćko and J. V. Milanović, “Forecasting demand flexibility of aggregated residential load using smart meter data,” *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 5446–5455, Sep. 2018.
[21] Ontario Energy Board. *Electricity Rates*. Accessed: Feb. 25, 2019. [Online]. Available: https://www.oeb.ca/rates-and-your-bill/electricity-rates
[22] GAMS Development Corporation. *General Algebraic Modeling System (GAMS)*. Accessed: Feb. 25, 2019. [Online]. Available: http://www.gams.com
[23] Government of Canada. *Historical Climate Data*. Accessed: Feb. 25, 2019. [Online]. Available: http://climate.weather.gc.ca

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